

THE DETERMINATION OF COEFFICIENTS OF DISCHARGE AND FLOW EQUATIONS  
FOR AIR FLOW THROUGH SOME PARTICULAR AN STANDARD FITTINGS AT  
ELEVATED TEMPERATURES CONSIDERING PRESSURE RATIO AS  
THE ONLY INFLUENCE FOR EACH GEOMETRY


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In Partial Fulfillment  
of the Requirements for the Degree  
Master of Science in Aeronautical Engineering

By  
John A. Bennett  
July 1956

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## NOTATION

## ENGLISH

$a$	fitting flow area, sq. ft.
$A$	fitting flow area, sq. in.
$a_o$	pipe flow area on either side of fitting, sq. ft.
$B$	system geometry constant, 1/sec.
$C$	constant
$d$	ordinary differential operator
$D_p$	pipe inside diameter on either side of fitting, in.
$E$	thermal expansion factor
$f$	function
$f_1$	function
$g$	function; or acceleration of gravity, 32.17 ft. per sec. per sec.
$K$	coefficient of discharge
$K_o$	coefficient of discharge for incompressible flow
$n$	ratio of specific heats, 1.4 for air
$p$	applied or input pressure as a function of time, psia
$p_1$	internal pressure as a function of time, psia
$P_1$	pressure upstream of fitting, psfa
$P_2$	pressure downstream of fitting, psfa
$P_1$	pressure upstream of fitting, psia
$P_2$	pressure downstream of fitting, psia
$\Delta p$	$P_1 - P_2$ , psfa

$\Delta P$	$P_1 - P_2$ , psia
$r$	pressure ratio $P_2/P_1$
$R$	gas constant in the Equation of State for air
$t$	time, sec.
$T$	absolute temperature of air upstream to the minimum restriction of the system, $^{\circ}R$
$T_1$	absolute temperature of air upstream to the minimum restriction of the system, $^{\circ}R$
$T_i$	system internal air absolute temperature, $^{\circ}R$
$V$	chamber volume, cu. in.
$w$	weight mass flow of air, lbs. per sec.
$w_t$	theoretical weight mass flow of air, lbs. per sec.
$Y$	expansion factor for air

## GREEK

$\Phi$	mass flow factor, $w\sqrt{T}/\Delta P_1$ , $(^{\circ}R)^{1/2}$ per sec.
$\beta$	ratio of fitting inside diameter to pipe inside diameter
$\rho$	density, lbs. per cu. ft.



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## SUMMARY

This paper presents an empirical determination of coefficients of discharge for six AN standard fittings commonly used in pressure-sensing instrumentation. These coefficients were derived from experimental results (1) for temperatures ranging from approximately 70° F. to 500° F. by use of an analysis similar to ASME methods (2) for obtaining the discharge coefficients of nozzles and sharp-edged orifices. The six fittings are the AN 815-4 and AN 815-6 "straight-throughs," the AN 821-4 and AN 821-6 "elbows," and the AN 824-4 and AN 824-6 "tees." (3)

Since the effects of Reynolds number, diameter ratio, and pipe size did not influence appreciably the particular results analyzed (1), the discharge coefficients obtained for each geometry were essentially dependent on the ratio of the pressure upstream of the fitting to the pressure downstream of the fitting. By the use of a curve-matching process, equations which relate coefficient of discharge to pressure ratio were derived for each type fitting. By a substitution of these expressions into a flow equation suggested by the ASME (2), the author obtained the relation of the mass flow factor  $\Phi$  to the pressure ratio  $r$  for each geometry.

From the resulting equations it was concluded that the discharge coefficient for a typical AN standard fitting is approximately a linear function of pressure ratio for values of pressure ratio higher

than the critical value of 0.528. For values of pressure ratio lower than the critical value, the discharge coefficient may be approximated by a second-degree function of pressure ratio. The above results are valid for temperatures ranging from approximately 70° F. up to 500° F.

The discharge coefficient for sharp-edged orifices was shown in Ref. 8 to be proportional to  $\sqrt{1 + \text{pressure ratio } r}$  for  $r \geq 0.528$ , and approximately a linear function of pressure ratio for  $r \leq 0.528$ . These results, obtained for room temperatures, were extended to include temperatures up to 500° F. since the curve describing variation of mass flow factor with pressure ratio obtained by Ray in Ref. 1 agrees closely with Perry's curve in Ref. 8.

## CHAPTER I

## INTRODUCTION

General. --The high velocities obtained by present-day test vehicles, airplanes, and missiles necessitates accurate predictions of the response, or lag, of pressure measuring instrumentation in these vehicles. Because of the more rapid pressure changes associated with these high velocities, the lag of a given barosensing system must be calibrated to a higher degree of accuracy than required at lower speeds in order to insure reasonably accurate instantaneous measurements.

Linear treatment of pressure lag. --Several papers (4), (5), and (6) have treated the problem of pressure lag by a transformation of the linear differential equation describing the response of a resistance-capacitance electrical system into an analagous pressure lag equation. In subsequent development, electrical resistance is treated as a function of viscosity, diameter of plumbing, and length of plumbing leading to the chamber which represents the measuring instrument. Electrical capacitance is considered a function of the internal pressure and the volume of the instrument. The solution of this equation makes possible the determination of a time constant from experimental tests in which a step input in pressure is applied. This time constant may then be multiplied by the applied rate of change of pressure to predict the pressure lag. From experimental tests conducted by the Sandia Corporation it was discovered, however, that the time constant determined by this method is dependent on the magnitude of the pres-

sure step.

Non-linear treatment of pressure lag.--Vaughn (7) arrived at the conclusion that the linear equation could not be used if the time constant were not in actuality a constant. This reasoning led to the development of a non-linear theory based on an empirical equation obtained by Perry (8) which describes the mass flow of air through a sharp-edged orifice as a function of the pressure ratio across the orifice. By application of this theory, Vaughn then showed that the time constant for linear theory actually approaches zero as the step pressure input approaches zero. From this result it was concluded that the application of a linear differential equation to the problem of pressure lag in pressure measuring systems could lead to very large errors. The more rigorous non-linear analysis should therefore be used for a system in which very rapid pressure fluctuations are expected.

In the non-linear theory (7), the basic equation, describing the response of the internal pressure  $p_1$  to the input pressure  $p$ , is

$$\frac{d p_1}{d t} = \frac{T_1}{T} p B \sqrt{\left| 1 + \frac{p_1}{p} \right|} \sqrt{|p - p_1|}$$

where

$d$  = ordinary differential operator

$t$  = time, sec.

$T_1$  = system internal air absolute temperature,  $^{\circ}R$

$T$  = absolute temperature of air upstream to the minimum restriction of the system,  $^{\circ}\text{R}$

$$B = \frac{K_o R A \sqrt{T}}{g V}$$

and

$K_o$  = basic coefficient of discharge

$R$  = gas constant in the Equation of State for air

$g$  = function; or acceleration of gravity, 32.17  
ft. per sec. per sec.

$A$  = fitting flow area, sq. in.

$V$  = chamber volume, cu. in.

In the sub-critical range the coefficient of discharge for the orifice is given by

$$K = K_o \sqrt{1 + \frac{P_i}{P}}$$

where  $K_o$  is constant for a given Reynolds number regime and upstream temperature. In general, the discharge coefficient  $K$  of a constriction is a function of the geometry of the constriction as well as the pressure ratio across the constriction. If the basic equation used were based on empirical relations defining the flow of air through the actual fittings in the system, rather than on an empirical equation for flow through an orifice, then more accurate prediction of the system pressure lag would be possible.

The flow characteristics of several AN standard fittings have now been investigated where Reynolds number was shown to have no appreciable effect on the flow (1). These were also performed with a small diameter ratio, the ratio of fitting diameter to the pipe diameter on either side of the fitting. The effects of the diameter ratio on discharge coefficients were therefore negligible. By an analysis of these experimental results, empirical equations will be derived which define the flow characteristics of these fittings for a temperature range from room temperature to 500° F. These equations may be of use in the refinement of the non-linear theory developed by the Sandia Corporation (7). It will be shown that the equations describing the flow characteristics of sharp-edged orifices at room temperature are valid for temperatures up to 500° F.

## CHAPTER II

### THEORY

General. --In order to proceed with the development of equations describing the flow through the six fittings to be analyzed, it is necessary first to define certain parameters which will be used in this development. Certain basic equations must also be presented and the assumptions necessary for the development of the theory must be stated.

The weight flow  $w$  through a circular constriction is a function of the flow  $A$  of the constriction, the Reynolds number  $R$  of the flow, the diameter ratio  $\beta$ , the pipe diameter  $D_p$ , the specific heat ratio  $n$ , the head pressure  $P_1$ , the back pressure  $P_2$ , and the absolute temperature  $T$ ; or

$$w = f(A, R, \beta, D_p, n, P_1, P_2, T) \quad (1)$$

Derivation of the mass flow factor. --For the particular tests being analyzed, Reynolds number effects are small, assuming the variation of mass flow with  $R$  is approximately the same for orifices and fittings. With this assumption, there would be a maximum variation of approximately only 2 per cent in the mass flow if Reynolds number were varied from the lower limit to the upper limit of the tests, holding all other parameters constant. The influence of diameter ratio and pipe diameter are generally considered negligible for  $\beta$  ratios less than 0.1. Since the



maximum  $\beta$  ratio for the tests was approximately 0.1, these effects were disregarded. It has been shown (1) for temperatures ranging from room temperatures up to  $500^{\circ}$  F. that the mass flow varies inversely as the square root of the absolute temperature. It was also proven that the mass flow varies directly with the flow area of the constriction. The specific heat ratio for air varies from a constant value by approximately only 2 per cent over the entire temperature range of the tests (9). The effect of a varying  $n$  is therefore assumed to be negligible. With the foregoing considerations, then

$$w = \frac{A}{\sqrt{T}} f(P_1, P_2) \quad (2)$$

By rearrangement, the equation becomes

$$\frac{w \sqrt{T}}{A} = f(P_1, P_2) \quad (3)$$

From experimental results (1), if the left-hand term is divided by  $p_1$ , then the factor obtained is a function of pressure ratio  $\frac{P_2}{P_1}$ , or

$$\frac{w \sqrt{T}}{A P_1} = f_1 \left( \frac{P_2}{P_1} \right) \quad (4)$$

If  $w \sqrt{T} / AP_1$  is defined as the mass flow parameter  $Q$ , and the pressure ratio  $P_2 / P_1$  is given the symbol  $r$ , then

$$Q = f(r) \quad (5)$$

Derivation of the basic flow equation. --The derivation of a mathematical expression describing the experimental results of Ray (1) is based on the equation for incompressible flow through a constriction (2). The effects of resistance, non-uniformity, and turbulence are considered negligible so that

$$w_t = \frac{1}{\sqrt{1 - \left(\frac{a}{a_0}\right)^2}} \quad a \sqrt{2 g \rho_1 \Delta p} \quad (6)$$

where

$w_t$  = theoretical weight mass flow, lbs. per sec.

$a$  = fitting flow area, sq. ft.

$a_0$  = pipe flow area on either side of fitting, sq. ft.

$\rho_1$  = density, lbs. per cu. ft.

If the approach and constriction cross-sectional areas are circular, this becomes

$$w_t = \frac{1}{\sqrt{1 - \beta^4}} \quad a \sqrt{2 g \rho_1 \Delta p} \quad (7)$$

If the pressure units are expressed in pounds per square inch and the area units in square inches, then

$$w_t = \frac{0.668 A}{\sqrt{1 - \beta^4}} \sqrt{\rho_1 \Delta P} \quad (8)$$

since  $\rho_1 = 2.70 P_1/T_1$  and  $\Delta P = P_1 (1-r)$ , then

$$w_t = 1.096 \frac{P_1 A}{\sqrt{T_1}} \frac{1}{\sqrt{1 - \beta^4}} \sqrt{1 - r} \quad (9)$$

Corrections to the basic flow equation. --The above equation, however, does not define the actual mass flow through an arbitrary constriction since, in deriving the equation, assumptions were made which are not valid for a real flow. The ASME (2) therefore makes three corrections to this formula, so that

$$w = K_o E Y w_t = 1.096 \frac{P_1 A}{\sqrt{T_1}} K_o E Y \sqrt{1 - r} \quad (10)$$

The basic coefficient of discharge  $K_o$  is a correction factor accounting for the effects of diameter ratio  $\beta$ , pipe size or roughness, and coefficient of viscosity. The correction  $E$  accounts for thermal expansion of the constriction due to high operating temperatures. The expansion factor  $Y$  is a correction for the compressibility of air. The thermal expansion factor is assumed to have a constant value of 1.000,

since there is a maximum deviation of only 0.6 per cent from this value for the tests being considered. Then in terms of the mass flow factor  $\Omega$ , the flow equation becomes

$$\Omega = 1.096 K_0 Y \sqrt{1-r} \quad (11)$$

If the term  $K_0 Y$  is given the symbol  $K$  and called the coefficient of discharge, then

$$\Omega = 1.096 K \sqrt{1-r} \quad (12)$$

## CHAPTER III

## PROCEDURE

General.--The experimental curves from which coefficients of discharge are evaluated are shown in Figs. 1, 2, and 3 for the AN 815, AN 821, and AN 824 fittings, respectively. From equation (5)  $\Omega$  is a function of  $r$  only, so that the flow equation may be expressed by

$$\Omega = f_1(r) = 1.096 K \sqrt{1-r} \quad (13)$$

The discharge coefficient  $K$  is therefore defined by the formula

$$K = \frac{\Omega}{1.096 \sqrt{1-r}} = \frac{f_1(r)}{1.096 \sqrt{1-r}} = g(r) \quad (14)$$

Evaluation of  $K$ . --Since the function  $f_1(r)$  is known from experiment,  $g(r)$  may be evaluated from equation (14) for a number of pressure ratios. This procedure is conducted in Tables (1), (2), and (3) for the AN 815, AN 821, and AN 824 fittings, respectively. Using the values from these tables, a curve representing the variation of flow discharge coefficient with pressure ratio is then plotted for each fitting. The curves for the three types of fittings are shown in Figs. 4, 5, and 6. By the use of a curve-fitting process as outlined in the appendix, a mathematical expression for  $K$  is obtained for each fitting. These ex-

pressions are presented in Chapter IV.

Determination of the flow equation in terms of  $Q$  and  $r$ . --By substitution of the appropriate mathematical expression for  $K$  in equation (13), the flow equation for each type fitting is obtained. The equations are listed in Chapter IV.

## CHAPTER IV

## RESULTS

Equations were developed for the coefficients of discharge of standard AN fittings at temperatures ranging from room temperature to 500° F. Also, the range of validity of the equations for coefficients of discharge of sharp-edged orifices (1) was extended to cover the range from room temperature to 500° F. These equations are summarized in tabular form as follows:

Equations for Coefficients of  
Discharge for AN Fittings

$$r \geq 0.528$$

Fitting	Equation
AN 815-4 AN 815-6	$K = 0.352 + 0.519 r$
AN 821-4 AN 821-6	$K = 0.310 + 0.298 r$
AN 824-4 AN 824-6	$K = 0.362 + 0.398 r$

$r \leq 0.528$	
Fitting	Equation
AN 815-4	$K = 0.445 + 0.161 r + 0.344 r^2$
AN 815-6	
AN 821-4	$K = 0.346 + 0.130 r + 0.187 r^2$
AN 821-6	
AN 824-4	$K = 0.414 + 0.193 r + 0.201 r^2$
AN 824-6	

Equations for Coefficients of  
Discharge for Sharp-edged Orifices

$$r \geq 0.528$$

Equation

$$K = 0.425 \sqrt{1 + r}$$

$$r \leq 0.528$$

Equation

$$K = 0.410 + 0.220 r$$

By using the preceding equations for coefficients of discharge of standard AN fittings and of sharp-edged orifices as correction factors to the basic flow equation, the following flow equations were obtained:



## Flow Equations for AN Fittings

$$r \geq 0.528$$

Fitting	Flow Equation
AN 815-4	$Q = 1.096 (0.352 + 0.519 r) \sqrt{1 - r}$
AN 815-6	
AN 821-4	$Q = 1.096 (0.310 + 0.298 r) \sqrt{1 - r}$
AN 821-6	
AN 824-4	$Q = 1.096 (0.362 + 0.398 r) \sqrt{1 - r}$
AN 824-6	

$$r \leq 0.528$$

Fitting	Flow Equation
AN 815-4	$Q = 1.096 (0.445 + 0.161 r + 0.344 r^2) \sqrt{1 - r}$
AN 815-6	
AN 821-4	$Q = 1.096 (0.346 + 0.130 r + 0.187 r^2) \sqrt{1 - r}$
AN 821-6	
AN 824-4	$Q = 1.096 (0.414 + 0.193 r + 0.201 r^2) \sqrt{1 - r}$
AN 824-6	

## Flow Equations for Sharp-edged Orifices

$$r \geq 0.528$$

Flow Equation

$$Q = 0.465 \sqrt{1 - r^2}$$

$$r \leq 0.528$$

Flow Equation

$$Q = 1.096 (0.410 + 0.220 r) \sqrt{1 - r}$$

## CHAPTER V

## CONCLUSIONS

1. If Reynolds number effects and diameter ratio effects are negligible, then the mass flow factor  $\beta$  for the flow of air through a given geometry fitting is principally a function of the pressure ratio  $r$  across the fitting.

2. The expressions for the coefficients of discharge of various fittings may be used in the development of a non-linear theory for prediction of the pressure lag in pressure sensing instrumentation of high speed vehicles.

3. The data are applicable for the Reynolds number range from 10,000 to 380,000,  $\beta \geq 0.1$ , and temperatures ranging from room temperature to 500° F.

## CHAPTER VI

### RECOMMENDATIONS

1. The flow of air should be studied for a wider range of sizes and types of fittings.
2. The effects of Reynolds number and diameter ratio on the flow of air through fittings should be investigated.

## APPENDIX

## APPENDIX I

DETERMINATION OF EQUATIONS FOR COEFFICIENTS OF  
DISCHARGE FOR AN 815-4 AND AN 815-6 FITTINGS

Pressure ratio  $r \geq 0.528$ .---From the curve of Fig. 5 it is seen that the coefficient of discharge is apparently a linear function of the pressure ratio. Assume that

$$K = A + B r$$

where A and B are constants to be determined from Fig. 5. From Fig. 5 at  $r = 0.900$ ,  $K = 0.819$  so

$$0.819 = A + 0.900 B$$

or

$$A = 0.819 - 0.900 B$$

From Fig. 5 at  $r = 0.528$ ,  $K = 0.625$  so

$$0.625 = A + 0.528 B$$

or

$$0.625 = 0.819 - 0.900 B + 0.528 B$$

so

$$B = \frac{0.819 - 0.625}{0.900 - 0.528} = 0.519$$

Therefore

$$A = 0.819 - 0.900 + 0.519$$

or

$$A = 0.352$$

The final equation for  $r \geq 0.528$  is therefore

$$K = 0.352 + 0.519 r$$

Pressure ratio  $r \leq 0.528$ .--If it is assumed that variation of  $K$  with pressure ratio may be approximated by a quadratic function of  $r$ , then

$$K = A + B r + C r^2$$

where  $A$ ,  $B$ , and  $C$  are constants to be determined from Fig. 5. From Fig. 5 at  $r = 0$ ,  $K = 0.445$ , so

$$A = 0.445$$

From Fig. 5 at  $r = 0.528$ ,  $K = 0.626$ , so

$$0.626 = 0.445 + 0.528 B + 0.283 C$$

and

$$B = \frac{0.626 - 0.445}{0.528} - \frac{0.283 C}{0.528}$$

or

$$B = 0.343 - 0.528 C$$

From Fig. 5 at  $r = 0.300$ ,  $K = 0.524$ , so

$$0.524 = 0.445 + 0.300 B + 0.090 C$$

or

$$0.524 = 0.445 + 0.300 (0.343 - 0.528 C) + 0.090 C$$

and

$$= 0.524 + 0.445 + 0.1029 = 0.1596 C - 0.090 C$$

so

$$C = \frac{0.0239}{0.0696} = 0.344$$

Therefore

$$B = 0.343 - 0.528 (0.344) = 0.161$$

The final equation for K is therefore

$$K = 0.445 + 0.161 r + 0.344 r^2$$

## APPENDIX II

DETERMINATION OF EQUATIONS FOR COEFFICIENTS OF  
DISCHARGE FOR AN 821-4 AND AN 821-6 FITTINGS

Pressure ratio  $r \geq 0.528$ .--From the curve of Fig. 6 it is seen that the coefficient of discharge is apparently a linear function of the pressure ratio. Assume that

$$K = A + B r$$

where A and B are constants to be determined from Fig. 6. From Fig. 6 at  $r = 0.900$ ,  $K = 0.578$ , so

$$0.578 = A + 0.900 B$$

or

$$A = 0.578 - 0.900 B$$

From Fig. 6 at  $r = 0.528$ ,  $K = 0.467$ , so

$$0.467 = A + 0.528 B$$

or

$$0.467 = 0.578 - 0.900 B + 0.528 B$$

so

$$B = \frac{0.578 - 0.467}{0.900 - 0.528} = 0.298$$

Therefore

$$A = 0.578 - 0.900 (0.298)$$



or

$$A = 0.310$$

The final equation for  $r \geq 0.528$  is therefore

$$K = 0.310 + 0.298 r$$

Pressure ratio  $r \leq 0.528$ .--If it is assumed that variation of  $K$  with pressure ratio may be approximated by a quadratic function of  $r$ , then

$$K = A + B r + C r^2$$

where  $A$ ,  $B$ , and  $C$  are constants to be determined from Fig. 6. From Fig. 6 at  $r = 0$ ,  $K = 0.346$ , so

$$A = 0.346$$

From Fig. 6 at  $r = 0.528$ ,  $K = 0.467$ , so

$$0.467 = 0.346 + 0.528 B + 0.283 C$$

and

$$B = \frac{0.467 - 0.346}{0.528} - \frac{0.283}{0.528} C$$

or

$$B = 0.229 - 0.528 C$$

From Fig. 6 at  $r = 0.300$ ,  $K = 0.402$ , so

$$0.402 = 0.346 + 0.300 B + 0.090 C$$

or

$$0.402 = 0.346 + 0.300 (0.229 - 0.528 C) + 0.090 C$$

and

$$= 0.402 + 0.346 + 0.069 = 0.1596 C = 0.090 C$$

so

$$C = \frac{0.013}{0.0696} = 0.187$$

Therefore

$$B = 0.229 - 0.528 (0.187) = 0.130$$

The final equation for K is therefore

$$K = 0.346 + 0.130 r + 0.187 r^2$$

## APPENDIX III

DETERMINATION OF EQUATIONS FOR COEFFICIENTS OF  
DISCHARGE FOR AN 824-4 AND AN 824-6 FITTINGS

Pressure ratio  $r \geq 0.528$ .—From the curve of Fig. 7, it is seen that the coefficient of discharge is apparently a linear function of the pressure ratio. Assume that

$$K = A + B r$$

where A and B are constants to be determined from Fig. 7. From Fig. 7 at  $r = 0.900$ ,  $K = 0.720$ , so

$$0.720 = A + 0.900 B$$

or

$$A = 0.720 - 0.900 B$$

From Fig. 7 at  $r = 0.528$ ,  $K = 0.572$ , so

$$0.572 = A + 0.528 B$$

or

$$0.572 = 0.720 - 0.900 B + 0.528 B$$

so

$$B = \frac{0.720 - 0.572}{0.900 - 0.528} = 0.398$$

Therefore

$$A = 0.720 - 0.900 (0.398)$$

or

$$A = 0.362$$

The final equation for  $r \leq 0.528$  is therefore

$$K = 0.362 + 0.398 r$$

Pressure ratio  $r \leq 0.528$ .—If it is assumed that variation of  $K$  with pressure ratio may be approximated by a quadratic function of  $r$ , then

$$K = A + B r + C r^2$$

where  $A$ ,  $B$ , and  $C$  are constants to be determined from Fig. 7. From Fig. 7 at  $r = 0$ ,  $K = 0.414$ , so

$$A = 0.414$$

From Fig. 7 at  $r = 0.528$ ,  $K = 0.572$ , so

$$0.572 = 0.414 + 0.528 B + 0.283 C$$

and

$$B = \frac{0.572 - 0.414}{0.528} - \frac{0.283 C}{0.528}$$

or

$$B = 0.299 - 0.528 C$$

From Fig. 7 at  $r = 0.300$ ,  $K = 0.490$ , so

$$0.490 = 0.414 + 0.300 B + 0.090 C$$

or

$$0.490 - 0.414 = 0.300 (0.299 - 0.528 C) + 0.090 C$$

and

$$- 0.490 + 0.414 + 0.090 = 0.1596 C - 0.090 C$$

so

$$C = \frac{0.014}{0.0696} = 0.201$$

Therefore

$$B = 0.299 - 0.528 (0.201) = 0.193$$

The final equation for K is therefore

$$K = 0.414 + 0.193 r + 0.201 r^2$$

Table 1

Calculation of Coefficients of Discharge  
for AN 815 Fittings

$S$	$r$	$\sqrt{1-r}$	$K$
0.083	0.990	0.100	0.759
0.207	0.950	0.223	0.847
0.283	0.900	0.316	0.814
0.337	0.850	0.388	0.794
0.376	0.800	0.447	0.767
0.428	0.700	0.548	0.712
0.460	0.600	0.633	0.663
0.471	0.528	0.686	0.625
0.473	0.500	0.707	0.610
0.478	0.400	0.775	0.574
0.482	0.300	0.837	0.526
0.483	0.250	0.866	0.508
0.485	0.150	0.992	0.479
0.486	0.000	1.000	0.443

Table 2

Calculation of Coefficients of Discharge  
for AN 821 Fittings

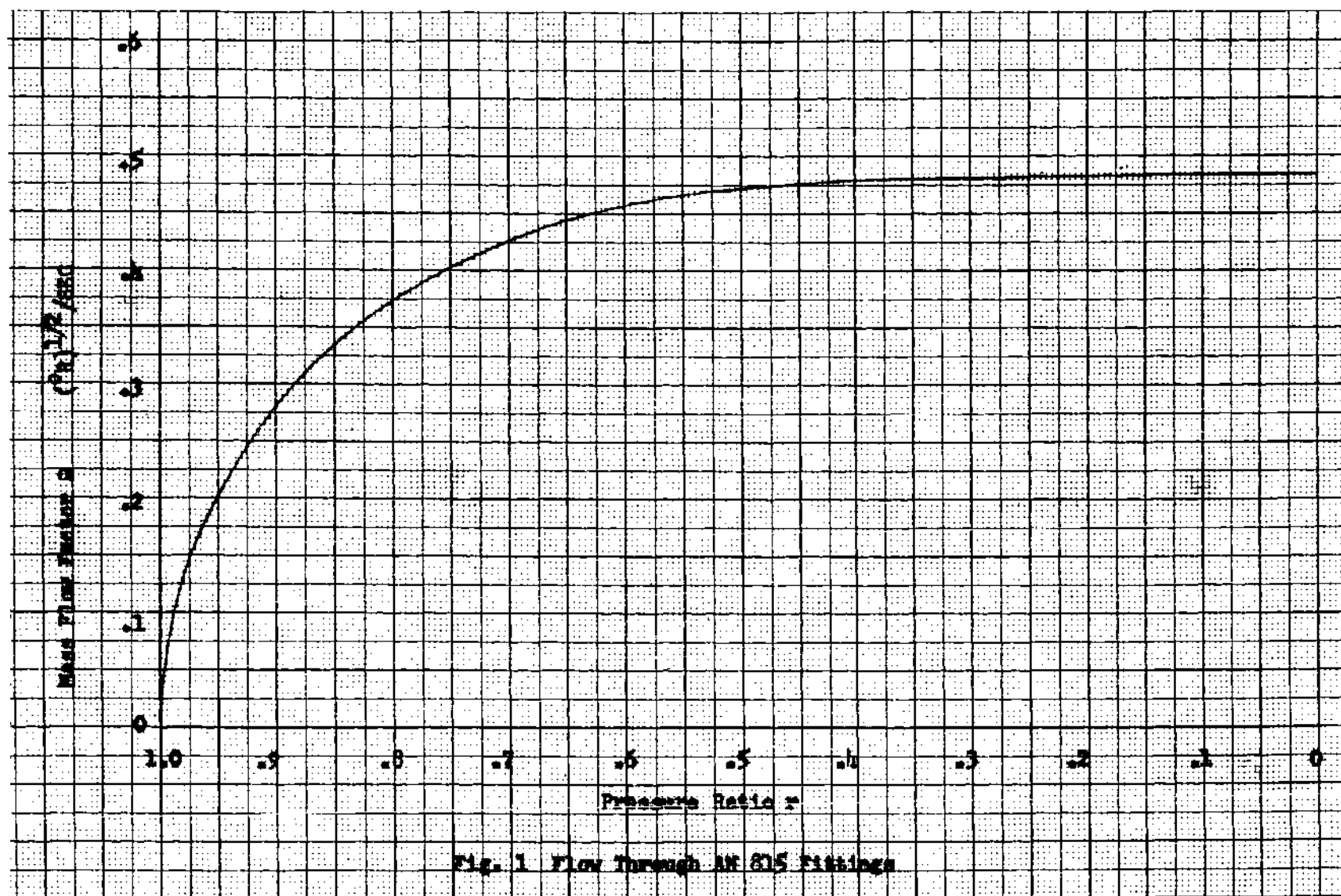
$\theta$	$r$	$\sqrt{1-r}$	$K$
0.055	0.990	0.100	0.503
0.140	0.950	0.223	0.573
0.200	0.900	0.316	0.578
0.239	0.850	0.388	0.562
0.268	0.800	0.447	0.547
0.311	0.700	0.548	0.518
0.339	0.600	0.633	0.490
0.351	0.528	0.686	0.467
0.356	0.500	0.707	0.452
0.364	0.400	0.775	0.429
0.370	0.300	0.837	0.403
0.372	0.250	0.866	0.392
0.375	0.150	0.922	0.371
0.378	0.000	1.000	0.345

Table 3

Calculation of Coefficients of Discharge  
for AN 824 Fittings

$\theta$	$r$	$\sqrt{1-r}$	$K$
0.078	0.990	0.100	
0.182	0.950	0.223	0.742
0.250	0.900	0.316	0.720
0.297	0.850	0.388	0.700
0.333	0.800	0.447	0.681
0.384	0.700	0.514	0.630
0.415	0.600	0.633	0.598
0.425	0.528	0.686	0.566
0.435	0.500	0.707	0.560
0.446	0.400	0.775	0.524
0.449	0.300	0.837	0.489
0.450	0.250	0.866	0.474
0.453	0.150	0.922	0.448
0.455	0.000	1.000	0.416





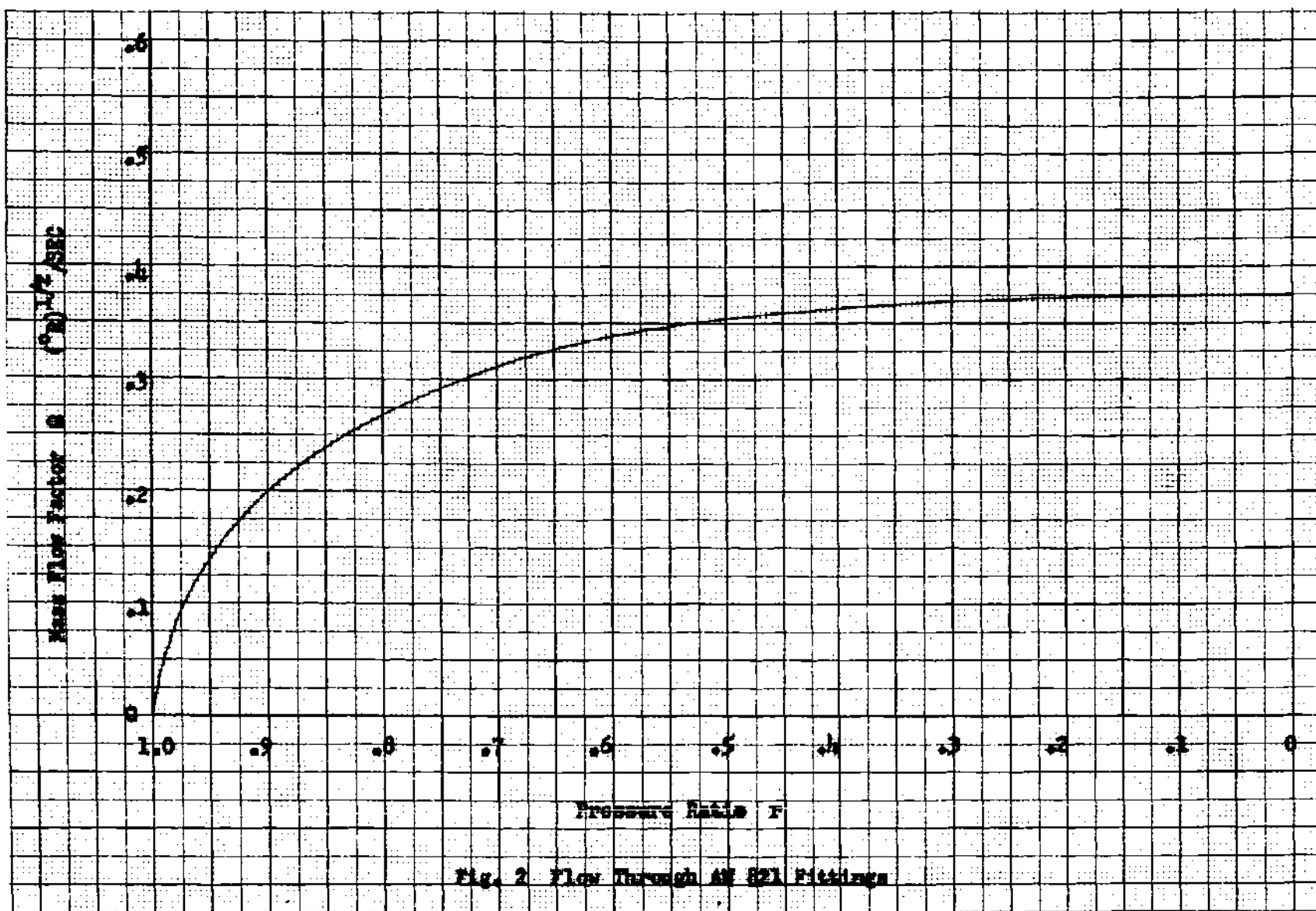
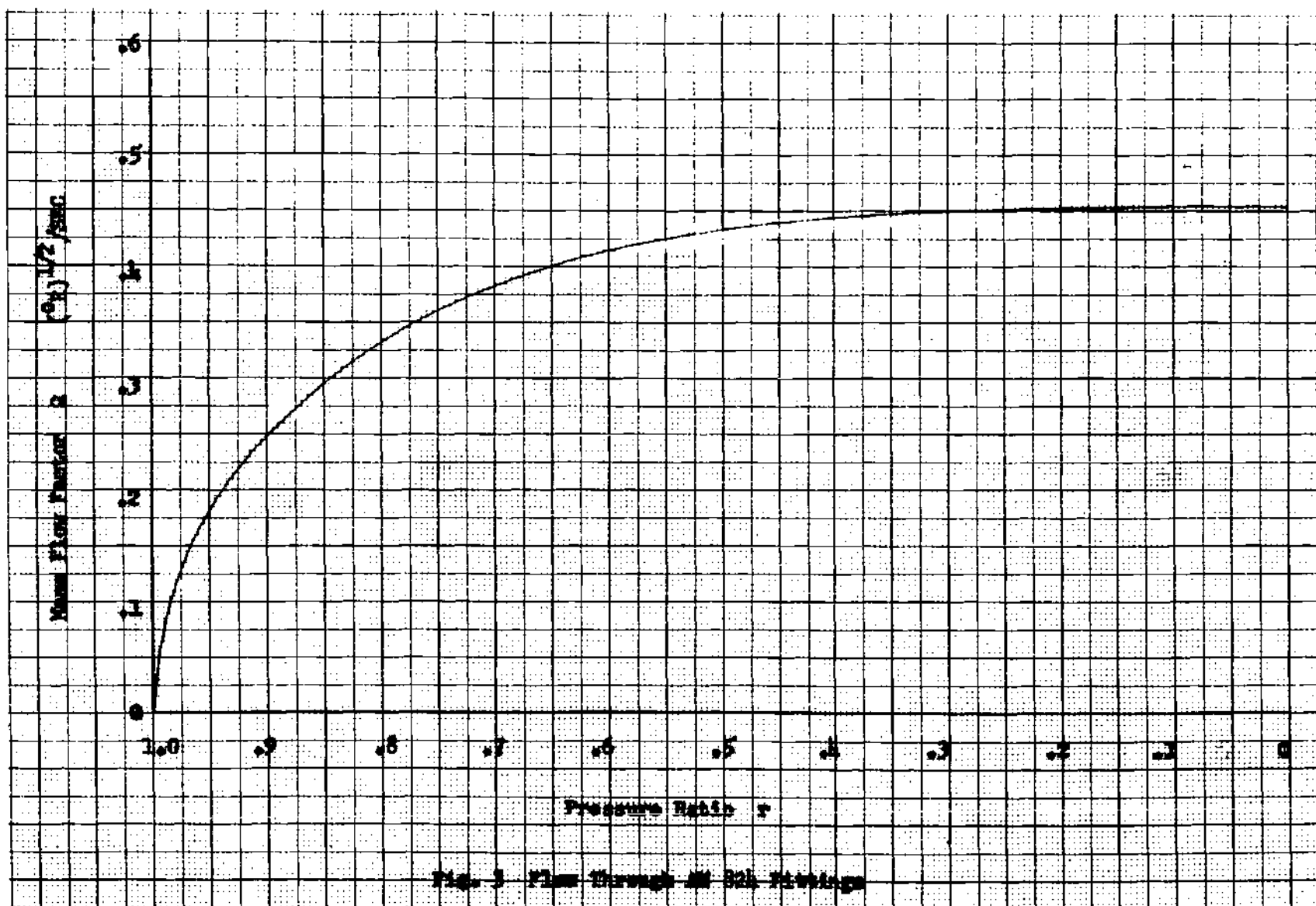


Fig. 2 Flow Through AW 521 Fittings



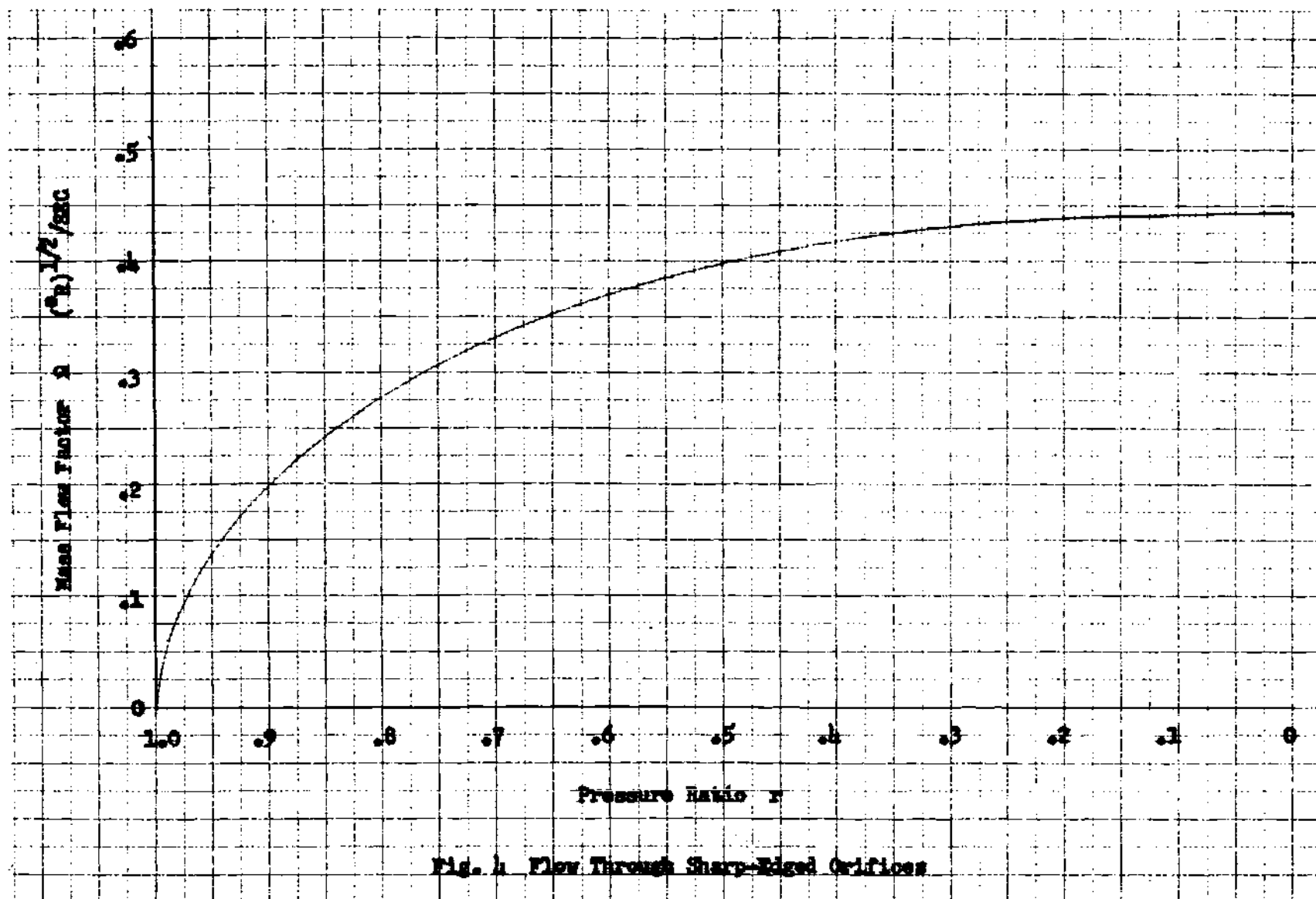
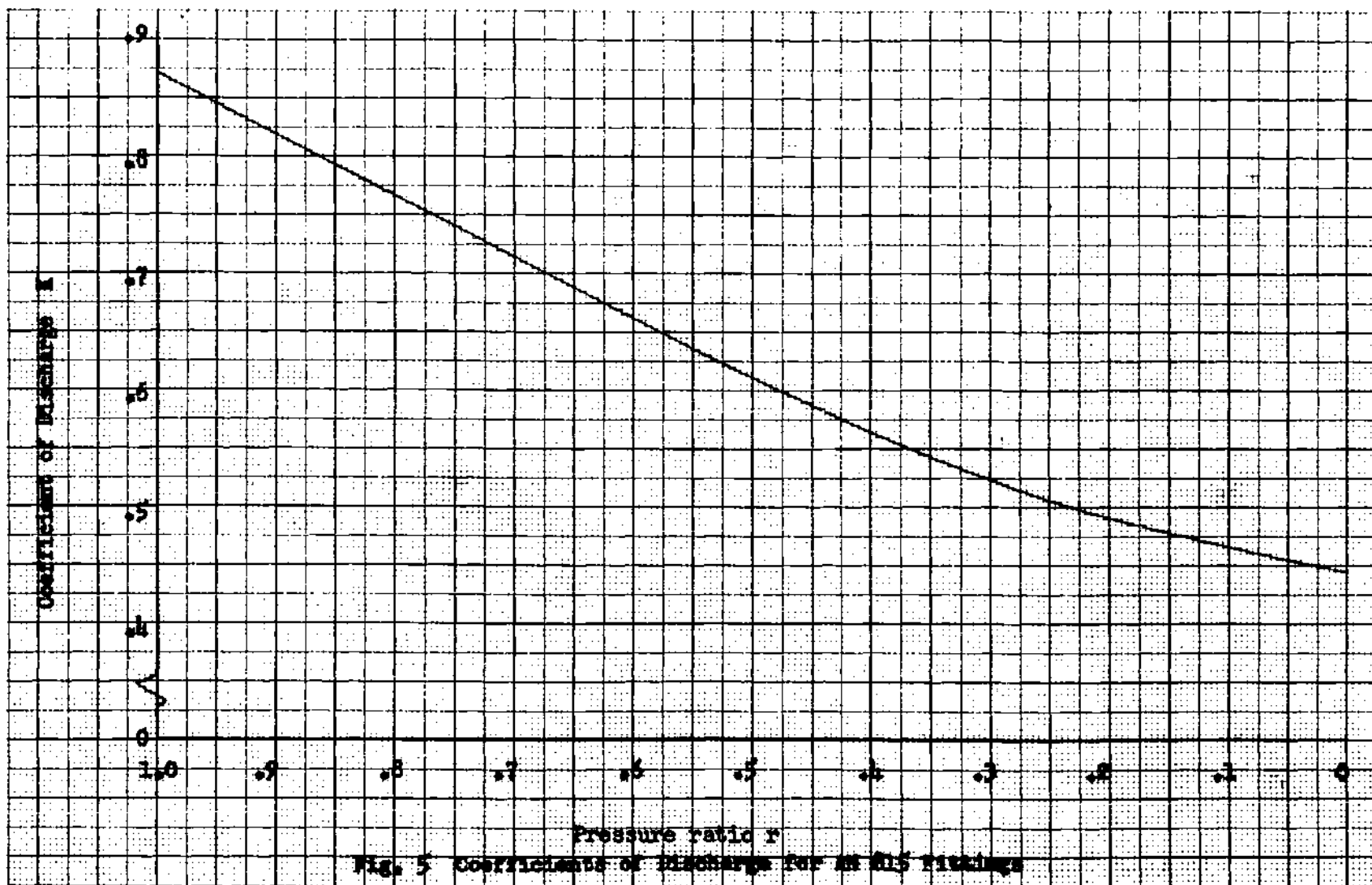
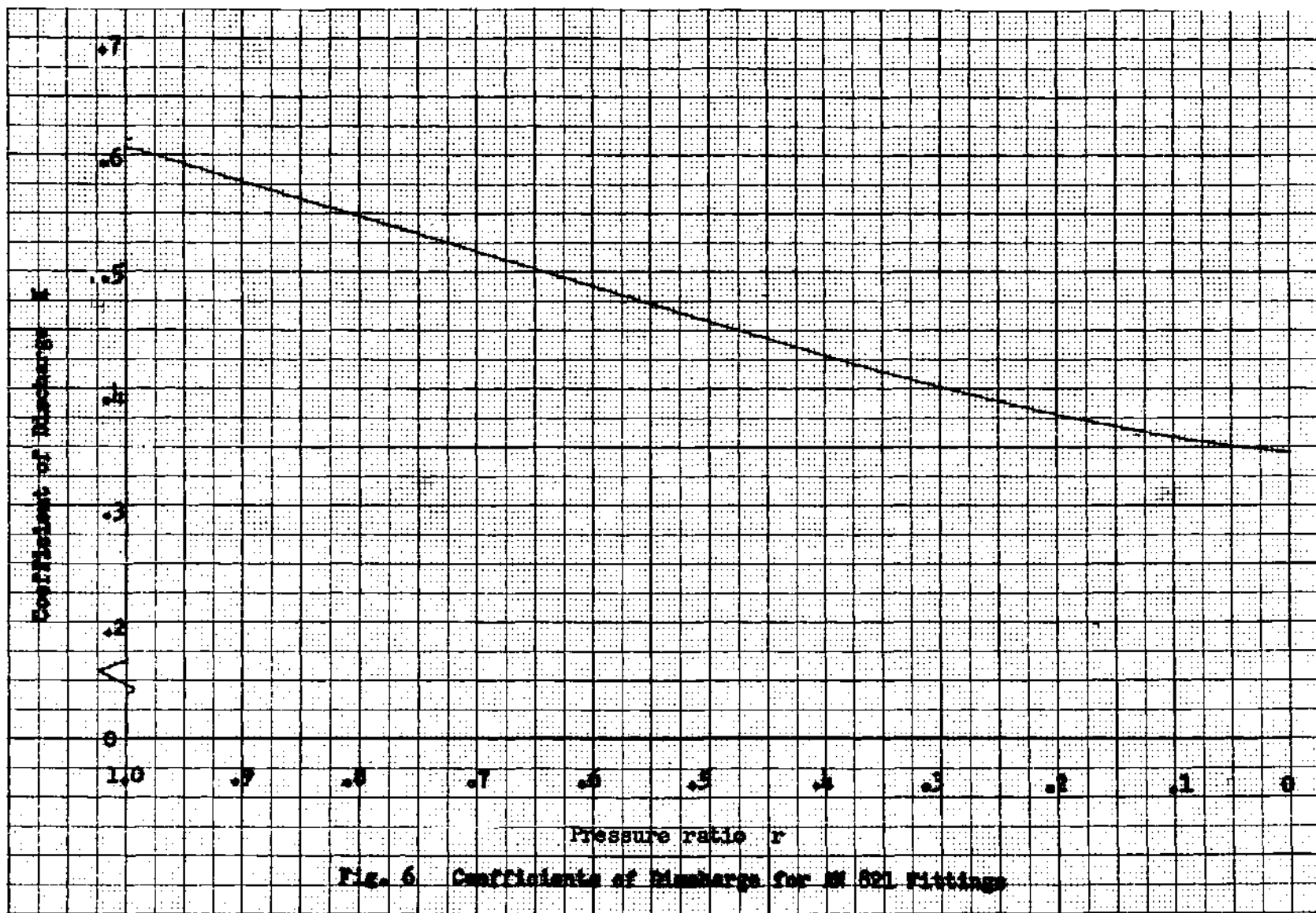
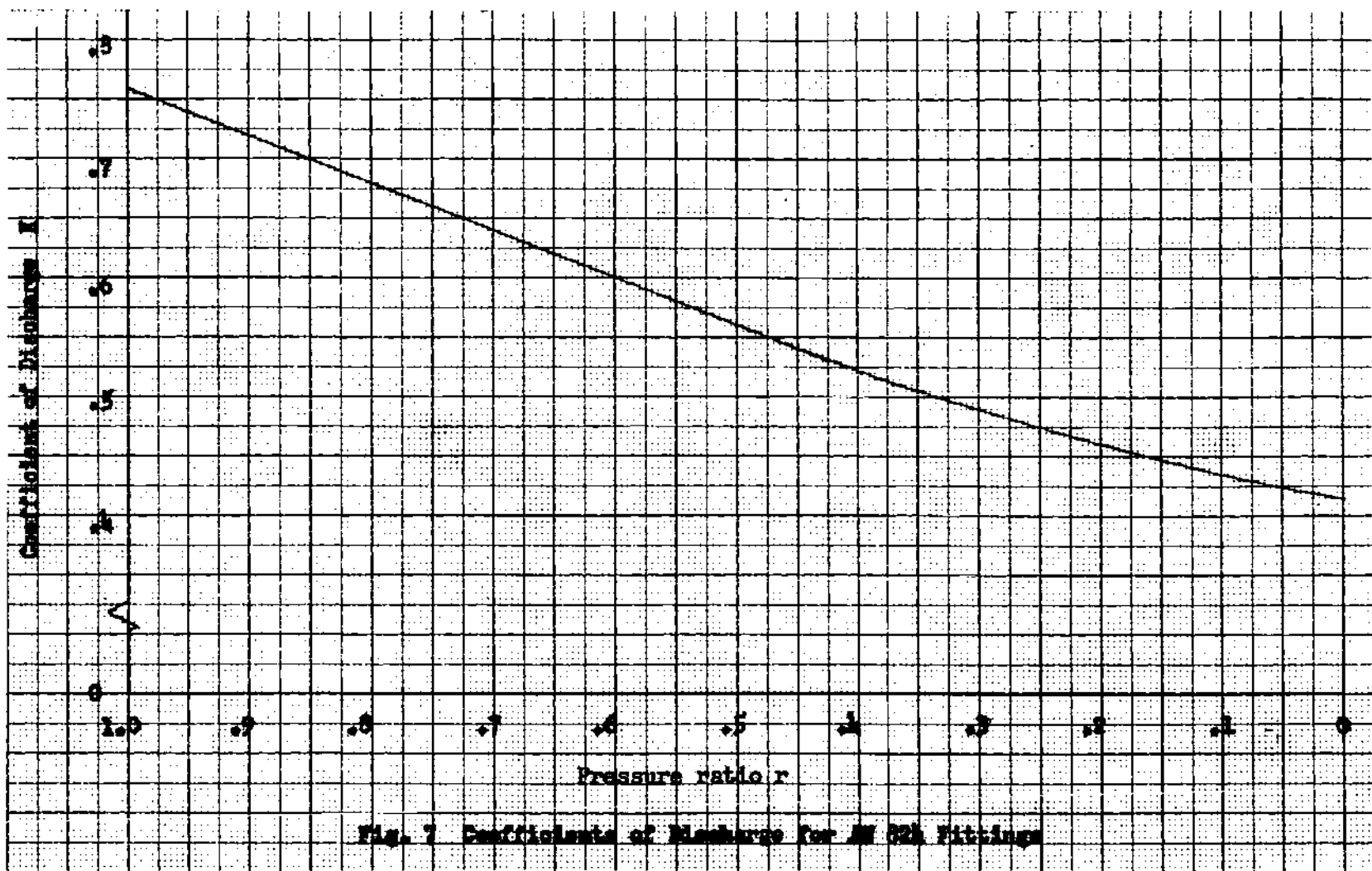


Fig. 1 Flow Through Sharp-Edged Orifices







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